# THE VERY NEAR FIELD
## THEORY, SIMULATIONS AND MEASUREMENTS
## OF SOUND PRESSURE AND PARTICLE VELOCITY IN THE VERY NEAR FIELD

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## Abstract

In acoustics concepts like the 'far field' and 'near field' are well known. The sound field at a position much closer to the source than its typical size is introduced in this paper as the "very near field". The boundary conditions are specified and verifying measurements are demonstrated. It is shown that in the very near field the particle velocity is frequency independent and almost independent of the distance to the source whereas the sound pressure is suppressed, frequency dependent and also almost independent of position.

## INTRODUCTION

In acoustics the properties of far field and near field are well known. In this paper the sound field very close to a source is described. We call this field the "very near field". Hydrodynamic nearfield and geometric nearfield might have a close resemblance to the very near field such as described in this paper.

Sensors to measure sound pressure are known, for the measurement particle velocity, a novel sensor is used, the so called Microflown [2], [3]. It is a very small sensor that measures the particle velocity directly and broad banded.

Particle velocity is a vector that can be measured in three directions. In this paper only the sound pressure and normal particle velocity are analyzed, in further research the lateral velocity will be investigated.

## THEORY

In any sound field the sound pressure $p$ and particle velocity $u$ are related by the specific acoustic impedance $Z$. In **the far field** this impedance is given by:

$$Z = \frac{p}{u} = \rho c \qquad (1)$$

Here $\rho$ is the density and $c$ is the speed of sound in the medium. The sound pressure and particle velocity are in phase and the sound pressure level (SPL) and particle velocity level (PVL) equal:

$$SPL \equiv 20Log\frac{p}{20\mu Pa} = 20Log\frac{\mathbf{u}\rho c}{20\mu Pa} \approx 20Log\frac{\mathbf{u}}{50nm/s} \equiv PVL \qquad (2)$$

In **the near field** the particle velocity level is elevated compared to the sound pressure level and a phase shift between sound pressure and particle velocity is observed. For example a point source has an acoustic impedance of:

$$Z = \frac{p}{u} = \rho c\frac{ikr}{1+ikr} \qquad (3)$$

with $k$ the wave number defined by: $k=2\pi f/c$ and $r$ the distance to the source, $f$ the frequency and $i=\sqrt{-1}$. As can be seen for reducing $r$, when the factor $kr$ becomes smaller that unity, the acoustic impedance drops and a phase shift occur. The PVL is elevated compared to the SPL:

$$PVL \equiv 20Log\frac{\mathbf{u}}{50nm/s} = 20Log\frac{p}{20\mu Pa}\sqrt{(1+\frac{1}{k^2r^2})} = SPL + 10Log(1+\frac{1}{k^2r^2}) \qquad (4)$$

More in general, the near field can be defined by the condition $kr<1$; so $r<c/2\pi f$. If $r$ is chosen infinitely small, Eq. (4) predicts that the particle velocity level should be infinite and this can of course not be true. If the source is not a (infinitely small) point source, and the distance to the source becomes very small compared to the size of the sound source, a region can be introduced that is called the **very near field**. In the very near field sound pressure and particle velocity are not very dependent on the distance to the source anymore.

### The Very Near Field

In this paragraph theory is presented that predicts the region of the very near field and how sound pressure and particle velocity behave in this region. A sound wave of frequency $f$ can be described by the acoustic potential $\varphi(\mathbf{r})$ obeying the Helmholtz equation:

$$\Delta\varphi + k^2\varphi = 0 \qquad (5)$$

Here $\Delta$ is the Laplace operator, $k=2\pi/\lambda=2\pi f/c$ is the wave number, $\lambda$ is the wavelength. To describe the sound field from some source, which is a vibrating surface, this equation should be solved with the boundary conditions:

$$\begin{aligned} &\frac{\partial \varphi}{\partial n} = v_n \text{ on the surface,} \\ &\varphi \propto \frac{\exp(ik_0 r)}{r} \text{ at infinity } r \to \infty, \end{aligned} \tag{6}$$

where $\partial/\partial n$ is the derivative normal to the surface, $v_n$ is the normal component of the velocity, in general, a function of the point on the vibrating surface. The observable acoustic values, sound pressure $p$ and particle velocity $\mathbf{v}$, are connected with the potential in the following way:

$$v = grad\, \varphi\ ,\ p = -i\omega\rho\varphi \tag{7}$$

where $\rho$ is the density of the background medium (air).

Suppose that the vibrating surface can be considered as flat on some lateral length scale $L$. One can choose the $x$-$y$ plane on the surface then the $z$ direction will coincide with the normal to the surface. In the direct vicinity of the surface the potential can be expanded in the power series in $z$:

$$\varphi(x,y,z) \approx \varphi_0(x,y) + \varphi_1(x,y)z + \frac{1}{2}\varphi_2(x,y)z^2 + O(z^3) \tag{8}$$

The functions $\varphi_{0,1,2}$ are connected with the velocities on the surface. From first the boundary condition (6) it follows that:

$$\varphi_1(x,y) = v_z(x,y,0) \equiv v_z^S \tag{9}$$

It means that measured particle very close the surface coincides with the surface velocity where the superscript S shows that the velocity should be taken on the surface (this is verified by measurement, see below). Connection of the velocity and potential gives us the relations:

$$\frac{\partial \varphi_0}{\partial x} = v_x^S\ ,\ \frac{\partial \varphi_0}{\partial y} = v_y^S \tag{10}$$

A comment should be made concerning these relations. The right hand sides are the velocity components taken directly on the surface. For any viscous fluid they should be zero. However, in most cases acoustic fields can be described with the ideal fluid equations. It means that the boundary layer where the velocity changes from zero to some finite value should be very thin. It is exactly the case because the thickness of boundary layer $\delta$ for the lateral velocity components is estimated as $\delta \sim \sqrt{\nu/\pi f}$, where $\nu$ is the fluid viscosity. For air and frequency $f=100$ $Hz$ this thickness is just of about of $50\mu m$. We have to consider the components $v_{x,y}^S$ as taken on the outer side of the boundary layer. Finally, taking the second derivative with $z$ from Eq. (8), one finds:

$$\varphi_2(x,y) = \left(\frac{\partial v_z}{\partial z}\right)^S \tag{11}$$

Therefore, the expansion Eq. (8) can be written as:

$$\varphi(x,y,z) \approx \varphi_0 + v_z^S z + \frac{1}{2}\left(\frac{\partial v_z}{\partial z}\right)^S z^2 \tag{12}$$

where $\varphi_0$ is also completely defined by the velocities by Eq. (10).
Let us insert now the potential (12) in the Helmholtz equation and take the limit $z \to 0$. We will find:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi_0 + k^2\varphi_0 + \left(\frac{\partial v_z}{\partial z}\right)^S = 0. \tag{13}$$

Suppose that the surface vibration can be described by some spatial wavelength $L$. It means that the surface vibrations can be represented by a harmonic in space function like $\sin(2\pi x / L)$ and similar for $y$-direction. The first two terms in Eq. (13) then can be estimated as:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi_0 \sim \left(\frac{2\pi}{L}\right)^2 \varphi_0, \quad k^2\varphi_0 = \left(\frac{2\pi}{\lambda}\right)^2 \varphi_0 \tag{14}$$

Typical situation in acoustic is that the vibrating body radiates the sound waves with the wavelength:

$$\lambda >> L \tag{15}$$

Then, according to (14), the second term in Eq. (5) is small in comparison with the first one and in the vicinity of the body the Helmholtz equation (5) is reduced to the Laplace equation:

$$\Delta\varphi = 0 \tag{16}$$

Equation (16) is the equation of uncompressible fluid that means that in this approximation the pressure level (dB) is negligible compared to the velocity level. The normal velocity in this range coincides with velocity of the vibrating surface.
Physically it means that nearby the surface the fluid can be considered as uncompressible and for this reason the sound pressure level (dB) is small compared to the particle velocity level. The latter has to be clearly explained. The sound waves are the compression-decompression waves and if Eq. (16) would be exact there would be no sound pressure at all. In reality this equation is approximate and the pressure still finite but it is suppressed. To find this suppression factor we have to estimate the value of $\varphi_0$. It

can be done using Eq. (10). As was explained above the derivative $\partial\varphi_0/\partial x \sim (2\pi/L)\varphi_0$ and similar for the derivative with respect to $y$. Nearby the surface all the velocity components are of the order of some vibration velocity $v_0$, which is defined by the vibration amplitude and frequency. One can use any suitable definition for $v_0$, for example, it can be defined as a maximal particle velocity $v_z$ measured nearby the surface. Then from Eq. (10) one gets the estimate:

$$\varphi_0 \sim \frac{L}{2\pi} v_0 \qquad (17)$$

The pressure is connected with the potential as $p=-2\pi i f \rho \varphi$ and in the limit $z \to 0$ one finds the following estimate for the pressure:

$$p \sim -i\rho c v_0 \frac{L}{\lambda} \qquad (18)$$

It shows that the pressure is suppressed in comparison with the particle velocity by the factor $L/\lambda$ (the ratio of the spatial wavelength $L$ of the source and the wavelength) and the phase between the particle velocity and sound pressure is shifted to 90 degrees.
Eq. (16) is true if the normal distance $r_n=z$ to the vibrating surface is small in comparison with the size $L$ and that the wavelength λ is larger than the vibrating surface $L$.

$$r_n << \frac{L}{2\pi} << \frac{\lambda}{2\pi} = \frac{c}{2\pi f} \qquad (19)$$

The condition (19) can be named by the condition of the ***very near field***. In this range the normal component of the velocity coincides with that for the vibrating surface but there are no restrictions on the lateral components of the velocity, which can be distributed in some way along the surface. In the table below the properties summarized.

| *Region* | *condition* | *u [m/s]* | *p [Pa]* | *Phase [deg]* |
|---|---|---|---|---|
| *Very near field* | $r_n << \frac{L}{2\pi} << \frac{\lambda}{2\pi}$ | $u(r_n) \approx constant$<br>$u(f) = constant$ | $p(r_n) \approx constant$<br>$p(f) \sim f$ | *80-90* |
| *Near field* | $\frac{L}{2\pi} << r_n << \frac{\lambda}{2\pi}$ | $u(r_n) \sim r^{-2}$ | $p(r_n) \sim r^{-1}$ | *80-10* |
| *far field* | $r_n >> \frac{\lambda}{2\pi}$ | $u(r_n) \sim r^{-1}$ | $p(r_n) \sim r^{-1}$ | *0-10* |

*Where $r_n$ is the normal distance to the source, L is the typical size of the source and λ is the wavelength of the sound wave.*

## SIMULATIONS AND MEASUREMENTS

A 19 cm circular aluminum plate was glued on a bass loudspeaker so that a piston was realized. The sound field (sound pressure and particle velocity) in front of this piston is simulated and measured as a function of the distance at several frequencies.
A half inch PU probe (Io Microflown element) of Microflown Technologies was used to measure the sound pressure and the particle velocity.

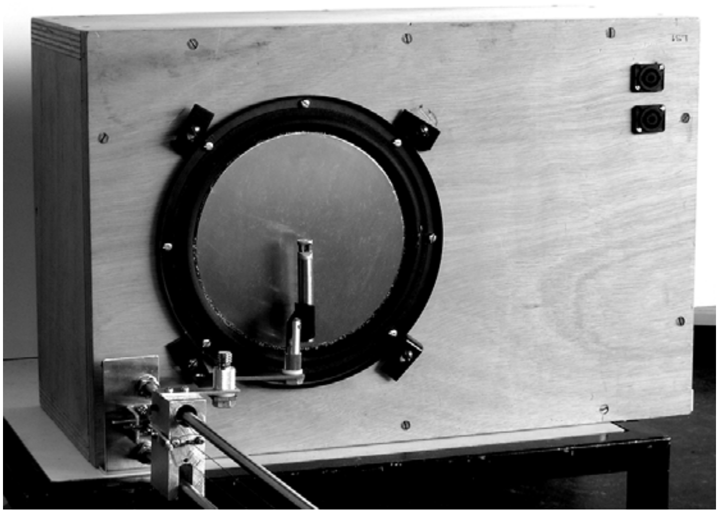

*Fig. 1: Measurement setup: a loudspeaker with an aluminum plate glued on and the PU probe on a traverse robot.*

The levels of sound pressure and particle velocity field for a plane circular piston in an infinite baffle along the axial distance are given by [1]:

$$p = \rho c \hat{\upsilon}_n e^{i(\omega t - kr)} \left( 1 - e^{-ik\left(\sqrt{r^2+L^2}-r\right)} \right)$$
$$\upsilon = \hat{\upsilon}_n e^{i(\omega t - kr)} \left( 1 - \frac{r}{\sqrt{r^2+L^2}} e^{-ik\left(\sqrt{r^2+L^2}-r\right)} \right) \qquad (20)$$

As can be seen in Fig. 1, at several frequencies the sound field is almost flat in the very near field; at a distance smaller than $L/2\pi$. In the near field ($L/2\pi<r<c/2\pi f$) the particle velocity has an elevated level compared to the sound pressure. The frequency dependent distance $r<c/2\pi f$, the distance where the sound field convert from the near field in to the far field is clearly noticeable. In the far field the sound pressure level and particle velocity level are of the same magnitude.

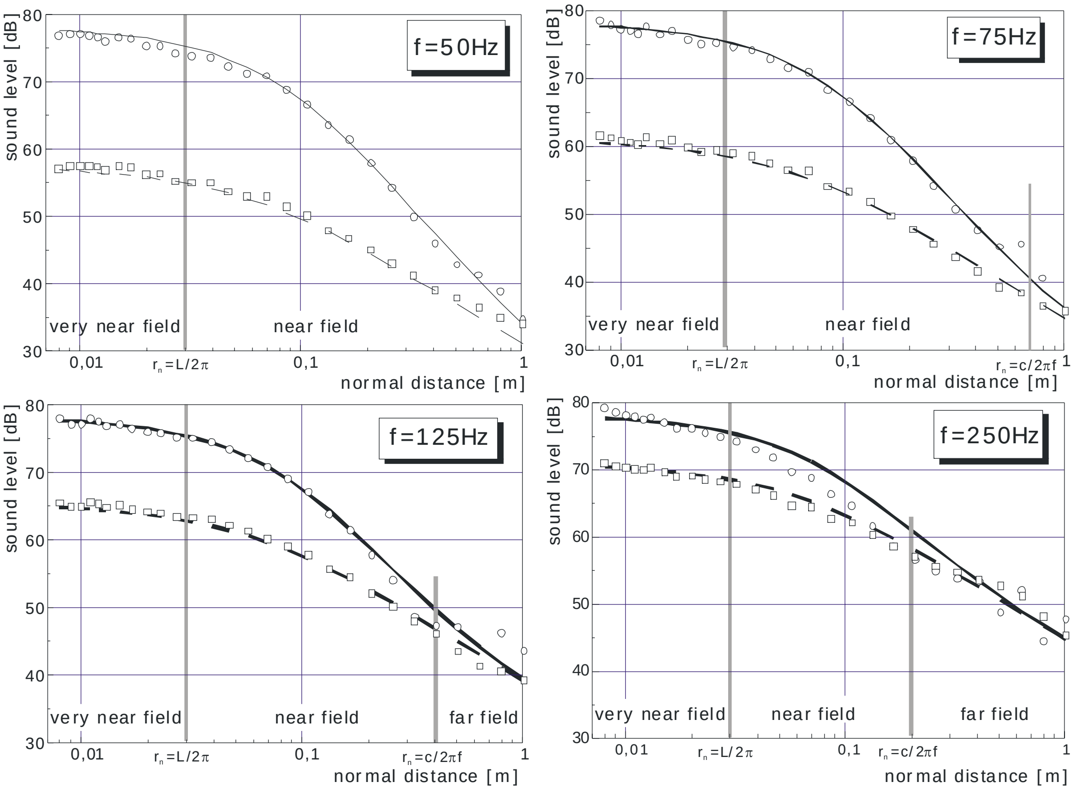


*Fig. 2: Simulated particle velocity level (black line), simulated sound pressure level (dashed line), measured particle velocity (dots) and measured sound pressure (squares) as function of normal distance to the source at different frequencies.*

## Laser vibrometer versus Microflown sensor at $r_n$=7mm

The particle velocity level of the sound field at 7 mm in front of the disk is compared with the surface velocity. The acoustic particle velocity level was measured with a Microflown and the surface velocity level was measured with a laser vibrometer (Polytec OFV505 sensitivity 25mm/s/V).

As can be seen in Fig. 3 (left), the surface velocity measured by the laser vibrometer and the particle velocity of the sound field measured at 7 mm in front of the plate measured with a Microflown sensor coincide. The deviation at 50Hz, 75Hz, 125Hz and 250Hz is determined respectively 0.5dB, 0dB, 0.5dB and -1.4dB.

The selfnoise, the signal that was measured when the plate was not excited (and the setup was placed on a stable underground), of both measurement techniques is comparable, see Fig. 3 (right). The selfnoise of the laser vibrometer is about 10dB higher than the selfnoise of the Microflown sensor. So the Microflown sensor performs 10dB (about 3 times) better. The noise level is measured in 1Hz bandwidth and given in dB with a reference level of 50nm/s (0dB equals 50nm/s) which is the pressure equivalent of the threshold of hearing at 1kHz.

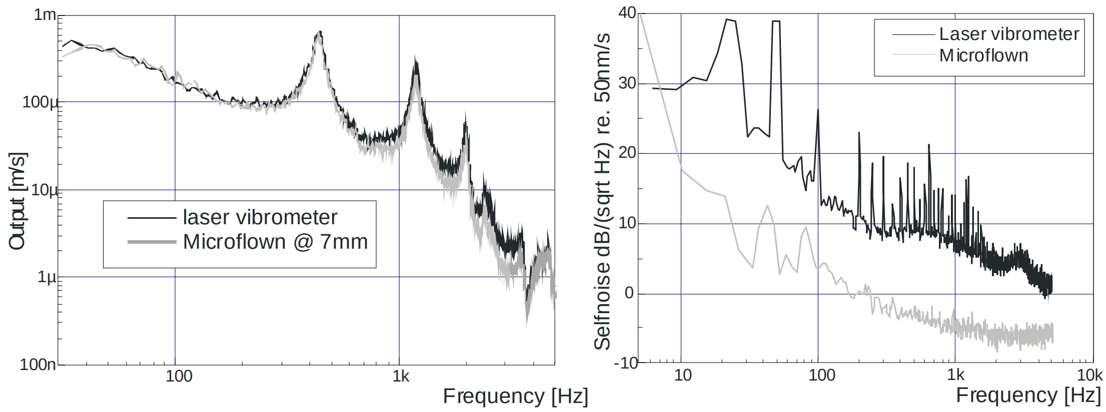


*Fig. 3:Left: measured particle velocity at 7mm distance from the plate and the surface velocity measured with a laser vibrometer. Right: Measured selfnoise of a Microflown sensor and the laser vibrometer 7mm from the non excited aluminum plate.*

## CONCLUSION

It has been shown (with a 19cm in diameter vibrating piston) that close to a vibrating object the Microflown can be used to measure the normal structural vibrations of that object. Only in the very near field the particle velocity level is only slightly dependent on the distance to the object and not of frequency. Conditions of the very near field are: the measurement distance should be closer than the structural size of the object divided by $2\pi$ and the wavelength of the sound should be larger than the structural size of the object.

## References

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