THE WHEATSTONE GADGET
A SIMPLE CIRCUIT FOR MEASURING DIFFERENTIAL RESISTANCE VARIATIONS

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ABSTRACT

This paper presents a simple circuit for measuring differential resistance variations. The Wheatstone Gadget, or The Gadget, is able to measure the same parameters as the Wheatstone Bridge in combination with an emitter-coupled pair [1], but with fewer components and fewer supply voltages. The Gadget is a simple circuit with a lot of possibilities, as shown in this paper. It needs only one power supply and is especially designed for small resistor values, which makes it well suitable for micromechanical applications.

INTRODUCTION

In many sensors a physical input quantity causes a variation in resistance, which has to be converted into an electrical output quantity. To increase the sensitivity and decrease undesirable side effects, most of the sensors are constructed in a way that the physical input quantity results in an increase in one resistance and a decrease in another.

An example is the *m-flown* [2], an acoustical flowsensor in which an acoustical flow causes a differential resistance variation. As in most micromechanical sensors the applied resistors are metalfilms because these can be manufactured with small tolerances. Typical values of metalfilm resistors in micromechanics are below one kΩ.

In this paper the following features of *The Gadget* will be presented: the transfer function, the common mode rejection ratio (CMRR), the power supply rejection ratio (PSRR), the offset tolerance and the signal to noise ratio (S/N). Furthermore some circuits based on *The Basic Gadget* will be presented.

THE BASIC GADGET

The Gadget uses only one power supply which can be chosen almost freely. It is basically a Widlar current mirror [1], the output current is a function of the input current and the emitter resistors. If the resistors have the same value the output current will be approximately the same as the input current.

The Basic Gadget is shown in Fig. 1 with at the left the circuit and at the right the block diagram. In this case it is designed with a set of two NPN transistors. The output current of *The Gadget* is given by:

$$I_{out} = I\frac{R_1}{R_2} - \frac{U_T}{R_2}\ln\left(\frac{I_{out}(\alpha_{FE1}+1)}{\left(I-\frac{I_{out}}{\alpha_{FE2}}\right)\alpha_{FE1}}\right) - \frac{I_{out}}{\alpha_{FE2}}\frac{(R_1+R_2)}{R_2} \quad (1)$$

U_T represents q/kT≈25mV. This transcendental equation cannot be solved analytically. If R_1⌚R_2 and $\alpha_{FE1}\approx\alpha_{FE2}>>1$ the last term (the DC offset term) can be neglected and eq. (1) simplifies to eq. (2).

$$I_{out} \approx I\frac{R_1}{R_2} - \frac{U_T}{R_2}\ln\left(\frac{I_{out}}{I}\right) \quad (2)$$

Under the linearity condition:

$$IR_1 >> U_T \ln\left(\frac{I_{out}}{I}\right) \qquad (3)$$

Finally eq. (1) simplifies to eq. (4). For small differential resistor variations, i.e. $\Delta R_1=-\Delta R_2$, and a nominal value $R_1=R_2=R$, the transfer function is eq. (5):

$$I_{out} \approx I\frac{R_1}{R_2} \quad ; \quad \Delta I_{out} = 2I\frac{\Delta R}{R} \qquad (4,5)$$

The signal to noise ratio is given by:

$$S/N = \frac{2\frac{\Delta R}{R} I}{\sqrt{BW\left(1.25x10^{-3}\frac{q\alpha_{FE}}{R^2} I^{-1} + \frac{4k}{R^2}(T_{bjt}R_{bb} + 2T_R R) + \frac{q}{10R} + 2qI\right)}} \qquad (6)$$

in which q is the charge of an electron, α_{FE} the forward current gain, R_{bb} is the base series resistance and T_{BJT} the temperature of the BJT. k is Boltzmann's constant, T_R the temperature of the sensing resistors and BW the bandwidth of the signal. Transistor noise models are used as in [1], the influence of 1/f noise is neglected here. In most cases the factor 2qI in eq. (6) is dominant in the denominator and the S/N-ratio will be proportional to the square root of I.

In contrast with the Wheatstone bridge used in combination with an emitter-coupled pair, the effect of a DC differential resistance on *The Basic Gadget* is not limited. The input voltage of an emitter-coupled pair is limited to approximately 50 mV [1].

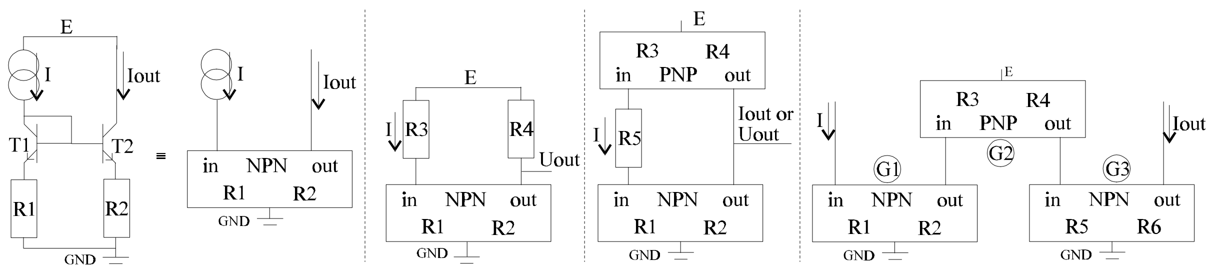

Fig. 1;2;3;4: The Basic Gadget; The Gadget; The Gadget Duo Sensation; The Cascade Gadget.

THE GADGET

Using the formulas of *The Basic Gadget* it is easy to find an expression for the transfer function of circuit in Fig. 2, the current source is simply implemented with resistor R_3 and the output current is converted to a voltage with resistor R_4.

The DC bias condition of *The Gadget* is (see also Fig. 1 and 2):

$$U_{out} = E - IR_4 > U_{E,T2} + U_{CE,min} \approx U_B \qquad (7)$$

The bias current is given by eq. (8). Substituting eq. (7) in eq. (8) the DC bias condition yields eq. (9).

$$I = \frac{E - U_{BE1}}{R_3 + R} \quad ; \quad R_3 \geq R_4 \qquad (8,9)$$

The transfer function of *The Gadget* using eq. (5) and eq. (8) is given by eq. (10):

$$Du_{out} = -2IR_4\frac{DR}{R} = -\frac{R_4(E - U_{BE1})}{R_3 + R}\cdot 2\frac{DR}{R} \qquad (10)$$

With $R_4=R_3$ an optimal transfer function for the gadget is achieved. The common mode transfer function ($\Delta R_1=\Delta R_2$) is given by eq. (11). The combination of eq. (10) and eq. (11) gives the CMRR:

$$\Delta u_{out,common} = \frac{I \cdot R \cdot R_4}{R_3}\frac{\Delta R}{R} \quad ; \quad CMRR = \left|\frac{\Delta u_{out,diff.}}{\Delta u_{out,common}}\right| \approx 2\frac{R_3}{R} \qquad (11,12)$$

If the power supply increases by ΔE, the current through R_3 increases with $\Delta I=\Delta E/(R_3+R)$, the current through R_4 increases by ΔI. The output voltage alters with ΔE minus $\Delta I \cdot R_4$. This leads to a PSRR of:

$$\frac{\Delta E}{\Delta U_{out}} \approx \left(1 - \frac{R_4}{R_3 + R}\right)^{-1} \qquad (13)$$

If the resistance R_4-$R_3 \approx R$, the effect of variations in the power supply will be minimalised. In this way it is possible to force a variation of the current by varying E without a variation of the output voltage eq. (8). Eq. (13) is only valid for $R_1=R_2$.

THE GADGET DUO SENSATION

The Gadget Duo Sensation (Fig. 3) consists of two *Gadgets.* It is capable of measuring two pairs of differentially varying resistors and can be used with voltage or current output. The current output is easy to calculate using eq. (1). When the nominal values of the resistors are equal ($R_1=R_2$ and $R_3=R_4$) the offset current will be eq. (14). The bias current is given by eq. (15):

$$I_{out,DC} \approx \frac{2(\alpha_{FE,NPN} - \alpha_{FE,PNP})}{\alpha_{FE,NPN}\alpha_{FE,PNP}} I \quad ; \quad I = \frac{E - 2U_{BE}}{R_1 + R_3 + R_5} \qquad (14,15)$$

Using $I = \frac{E - 2U_{BE}}{2R + R_C}$ eq. (5) the output current due to differential variations of the resistors is:

$$\Delta I_{out} = 2I\left(\left[\frac{\Delta R}{R}\right]_{NPN} - \left[\frac{\Delta R}{R}\right]_{PNP}\right) \qquad (16)$$

For voltage output the *PNP-Gadget* is an active load [1] for the *NPN-Gadget* and vice versa. If $I_{DC} \approx 0$ ($U_{out} \approx ½E$) the transfer function is:

$$\Delta U_{out} = \Delta I_{out}(r_{o,PNP} \| r_{o,NPN}) = \frac{U_{A,NPN} U_{A,PNP}}{U_{A,NPN} + U_{A,PNP}} \frac{\Delta I_{out}}{I} \qquad (17)$$

Using U_A as the Early voltage.

THE CASCADE GADGET

Because the output of a *Basic Gadget*, see eq. (5), has the same dimension and magnitude as the input of *The Basic Gadget* it is possible to cascade it. In Fig. 4 three *Gadgets* have been cascaded. The limiting factor for cascading is that the bias current will be reduced, last term of eq. (1). This limitation however can be overcome by injecting currents. If the resistors of each individual *Gadget* vary little and differentially and assuming $(\Delta R/R)^2 \approx 0$, the transfer function of *The Cascade Gadget* is:

$$I_{out} + \Delta I_{out} = I\prod_{i=1}^{N}\left(1 + 2\left[\frac{\Delta R}{R}\right]_{Gi}\right) \approx I + I\sum_{i=1}^{N} 2\left[\frac{\Delta R}{R}\right]_{Gi} \qquad (18)$$

Therefore it can be used as a summation circuit.

THE AM GADGET

Supplying *The Gadget* with an alternating voltage, superimposed on a DC voltage, (see Fig. 2) the output will generate an AM signal with suppressed carrier (called a *double-side band* signal [3]). The supply voltage of *The Gadget* is in this case:

$$E = E_0 [1 + a \cdot Cos(w_{HF} t)] \qquad (19)$$

using α (with α<1) as the ratio of the absolute value of the carrier and the DC voltage. The output of the *AM Gadget*:

$$\Delta u_{out} = 2IR_4 \cdot \beta \cos(\omega_{LF} t) + PSRR^{-1} \cdot \alpha E \cdot Cos(\omega_{HF} t) + IR_4 \alpha\beta\{Cos(\omega_{LF} t + \omega_{HF} t) + Cos(\omega_{LF} t - \omega_{HF} t)\} \qquad (20)$$

using β cos(ω_{LF}t) as ΔR/R. The PSRR, eq. (13), indicates how much the carrier will be suppressed.

SIMULATION AND MEASUREMENT RESULTS

The Gadget, see Fig 2. Parameters: R_3=820Ω; R_4=680Ω; ΔR/R=0.01; E=10V.

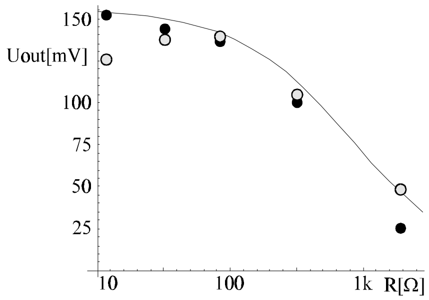

Fig. 5: Output of The Gadget as a function of R (the nominal value of R_1 and R_2). The line represents the theoretical model (eq. 10), the dots are PSPICE simulations, the open dots measurements

Two notes have to be made, 1, the decrease of the transferfunction can be avoided when a current source is used instead of the combination of R3 and a voltage source, 2, because of relative big ohmic losses for small resistor values, the measurements in this range are not as good as predicted.

The Gadget Duo Sensation has conveniently been used in our measurements [2]. One disadvantage of the voltage output is that the exact value of the Early voltage is unknown and thus the transfer function cannot be exactly determined.

The AM Gadget. Measurements have been performed with E=20V, α=0.71, β=8.6‰; R_3=3300Ω in series with a diode, R_4=3513Ω, $f_{LF,signal}$=200Hz, $f_{HF,carrier}$=10kHz. The model (eq. 20) and PSPICE both predict a factor ½α=2.8 reduction in the modulated side-band compared to the signal, measurements show a factor 4.9. A PSRR of 45dB (the carrier) and a minimum β of $7\cdot10^{-9}$ (u_{HF}=10μV_{RMS}) was measured at a side band.

FURTHER RESEARCH

To get a larger S/N ratio other types of transistors, the MOSFET and JFET, are being investigated. More variations of *The Gadget* are being examined. It is also possible to measure complex impedances like capacitors or inductances.

CONCLUSIONS

The Gadget is a simple circuit measuring differential varying resistances. It is easier to operate and has more tolerances than the Wheatstone bridge. For micromechanical applications it is well suitable, because it works well with low resistor values.

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