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THE UNCERTAINTY OF PURE TONE MEASUREMENTS IN REVERBERATION ROOMS BELOW THE SCHROEDER FREQUENCY

Finn Jacobsen

Acoustic Technology, Department of Electrical Engineering, Technical University of Denmark, Building 352, DK-2800 Kgs. Lyngby, Denmark
e-mail: fja@elektro.dtu.dk

Alfonso Rodríguez Molares

E.T.S.E. Telecomunicación, Universidade de Vigo, Campus Lagoas-Marcosende, E-36310 Vigo, Spain

Reverberation rooms are often used for measuring the sound power emitted by sources of sound. At medium and high frequencies where the modal overlap is high a fairly simple model based on sums of waves from random directions having random phase relations gives good predictions of the measurement uncertainty in such rooms. Below the Schroeder frequency the uncertainty is much larger, in particular if the source emits a pure tone. The established theory for this frequency range is based on ensemble statistics of modal sums and requires knowledge of mode shapes and the distribution of modal frequencies. This paper extends the far simpler random wave theory to low frequencies. The two theories are compared, and their predictions are compared with experimental and numerical results.

1. Introduction

It has been known for many years that the sound pressure at a given position in a reverberant room driven with a pure tone varies significantly with the frequency,[1] and that the sound pressure varies significantly with the position if the room is driven at a given frequency.[2] It is also well established that the sound power emitted by a pure-tone source in a reverberation room depends on the size and damping of the room and the position of the source, in particular below the Schroeder frequency.[3] Above the Schroeder frequency a fairly simple stochastic theory based on sums of coherent waves arriving from random directions and having random phases gives reliable predictions of the spatial fluctuations of the sound pressure.[2] The random wave theory requires very little information about the particulars of the room. An alternative theory is also stochastic in nature but based on sums of modes with random distributions of the modal frequencies.[3,4] The modal theory, which is more complicated than the random wave theory and requires more information about the room, is generally regarded as valid also below the Schroeder frequency, if only approximately.[4] However, there is surprisingly little experimental evidence of its validity in this frequency range.

This paper attempts to extend the simpler theoretical approach to low frequencies. Considerations on the statistics of kinetic and total energy density are also presented. The two theories are compared with experimental and numerical results.

2. A brief description of the existing theoretical models

The random wave theory is essentially due to Schroeder,[1] Andres,[5] Waterhouse,[2] and Lubman.[6] The alternative modal theory is essentially due to Lyon[3] and Davy,[4,7] but later modified so as to take account of findings by Weaver.[8]

2.1 The random wave theory

A harmonic sound field in a reverberation room is modelled as a sum of waves,

$$p(\mathbf{r}) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} A_n \mathrm{e}^{-\mathrm{j}\mathbf{k}_n \cdot \mathbf{r}}, \tag{1}$$

where $p(\mathbf{r})$ is the sound pressure at position $\mathbf{r}$, A_n is a complex random amplitude the phase angle of which is uniformly distributed between 0 and 2π, and $\mathbf{k}_n$ is a random wavenumber vector with a uniform distribution over all angles of incidence. Note that *no* information about the particulars of the room has entered into this stochastic pure-tone diffuse field theory at this stage. It is easy to show that the corresponding expression for the mean square pressure is a sum of two independent squared Gaussian variables with zero mean (random sums).[2,9] A sum of two squared Gaussian variables with zero mean has a chi-square distribution with two degrees of freedom,[10] from which it follows that the relative ensemble variance is 1,

$$\varepsilon^2 \left\{ p_{\mathrm{rms}}^2 \right\} = 1. \tag{2}$$

Jacobsen extended Waterhouse's results to particle velocity components and showed that the relative ensemble variance of both the kinetic energy density and the total energy density is three times less than the relative variance of potential energy density, that is,

$$\varepsilon^2 \left\{ w_{\mathrm{kin}} \right\} = \varepsilon^2 \left\{ w_{\mathrm{tot}} \right\} = \frac{1}{3}, \tag{3}$$

simply because the three perpendicular components of the mean square particle velocity are independent random variables in combination with the fact that the average kinetic energy density is identical to the average potential energy density.[9] Equation (2) has been validated experimentally many times, e.g., in Refs. 2 and 11. Because of lack of appropriate equipment Jacobsen was not able to validate Eq. (3) in the 1970s; this was done much later by Moryl and Hixson.[12]

About thirty years ago Jacobsen[9] and Pierce[13] independently used Eq. (1) to calculate the relative ensemble variance of the real part of the radiation impedance experienced by a monopole, and thus the sound power output of the monopole. They showed that the reverberant part of the sound field expressed by Eq. (1) because of the resulting random contribution to the radiation impedance gives rise to a relative variance of the sound power output of

$$\varepsilon^2 \left\{ P_{\mathrm{a}} \right\} = \frac{1}{M_{\mathrm{s}}}, \tag{4}$$

where M_{s} is the statistical modal overlap of the room (the product of the modal density and the statistical bandwidth of the modes). The modal overlap, which is proportional to the square of the frequency and the total absorption area in the room, is large above the Schroeder frequency, and therefore the sound power emitted by a monopole essentially equals its free field sound power.[9,13]

2.2 The modal theory

The original version of the alternative theory, due to Lyon,[3] is based on the analytical Green's function in a rectangular room, which is a modal sum.[13] Lyon assumed that the modal frequencies have a Poisson distribution (i.e., are distributed independently of each other) and predicted the ensemble variance of the sound power output of a monopole and the ensemble variance of the mean square pressure. Some years later Davy extended Lyon's theory by deriving a more general expression of the power transmission function averaged over multiple source and receiver positions assuming a 'nearest neighbour' distribution of the modal frequencies.[4] The assumption of the modal frequencies having a nearest neighbour distribution rather than a Poisson distribution came from evidence of a 'spectral rigidity' effect already discussed by Lyon;[3] the distribution of the modal frequencies is closer to the average density than one might have expected.[8] Finally in 1990 Davy modified his theory so as to take account of a modal frequency spacing described by the Gaussian orthogonal ensemble theory,[14] which is now generally accepted.[15] The resulting expression for the relative ensemble variance of the sound power of a monopole became

$$\varepsilon^2\left\{P_\mathrm{a}\right\} = \frac{K-1}{M_\mathrm{s}}, \tag{5}$$

where $K = E\left\{\psi^4\right\}/E^2\left\{\psi^2\right\} = \left(3/2\right)^3$ for oblique modes in a rectangular room (ψ is the mode shape (a product of cosines in a rectangular room)). With this value of K Eq. (5) is similar to but somewhat larger than the prediction given by Eq. (4) ($\varepsilon^2\left\{P_\mathrm{a}\right\} \simeq 2.38/M_\mathrm{s}$). The corresponding expression for the relative ensemble variance of the mean square pressure became

$$\varepsilon^2\left\{p_\mathrm{rms}^2\right\} = 1 + \frac{K^2-3}{M_\mathrm{s}}, \tag{6}$$

which asymptotically approaches Eq. (2) for high values of the modal overlap. (With $K = \left(3/2\right)^3$ Eq. (6) becomes $\varepsilon^2\left\{p_\mathrm{rms}^2\right\} \simeq 1 + 8.4/M_\mathrm{s}$.) Although there is little doubt that there is an increase in the variance of the mean square pressure in a frequency region below the Schroeder frequency compared with the asymptotic value of unity at high modal overlap,[16] there is, for some reason, surprisingly little, if any, experimental evidence in direct support of Eq. (6) in room acoustics.

It should be mentioned that other authors have suggested lower values of K. Weaver[8] and Lobkis *et al.*[17] are in favour of a value of 3; Langley and Brown have suggested a value of 2.7.[18]

2.3 Extension of the random wave theory

Recently Jacobsen and Rodríguez Molares[19] modified Eq. (4) by taking account of the local increase of the reverberant part of the sound field at the source position due to 'weak Anderson localisation' as predicted by Weaver and Burkhardt.[20] Equation (4) now be became

$$\varepsilon^2\left\{P_\mathrm{a}\right\} = \frac{F(M_\mathrm{s})}{M_\mathrm{s}}, \tag{7}$$

where the 'concentration factor' $F(M_\mathrm{s})$ is a function that goes smoothly from 3 to 2 as the modal overlap is increased.[15] The modified expression was validated experimentally in various reverberation rooms as well as by numerical calculations.[19] As pointed out in Ref. 19 it is interesting and somewhat surprising that the modal overlap enters into a theory that does not make use of the concept of modes. Since Eq. (7) and Eq. (5) are very similar (with F = 2 and $K = \left(3/2\right)^3$) these results also validated the modal prediction. With F = 2 and K = 3 the two expressions are identical.

The finite ensemble variance of the sound power emitted by the source means that the ensemble average of $|A_n|^2$ varies from outcome to outcome of the stochastic process. Such variations are not taken into account by Eq. (2). Thus, one might expect Eq. (2) to become

$$\begin{aligned}\varepsilon^2\left\{p_\mathrm{rms}^2\right\} &= \varepsilon^2\left\{x\left(1+y\right)\right\} = \frac{E\left\{x^2(1+y)^2\right\}}{E^2\left\{x\left(1+y\right)\right\}} - 1 = \frac{E\left\{x^2\right\}E\left\{\left(1+y\right)^2\right\}}{E^2\left\{x\right\}E^2\left\{\left(1+y\right)\right\}} - 1 \\ &= 2\left(1+E\left\{y^2\right\}\right) - 1 = 1 + \frac{2F\left(M_\mathrm{s}\right)}{M_\mathrm{s}},\end{aligned} \tag{8}$$

where x is the original mean square pressure (an exponentially distributed variable) and y is the real part of the reverberant part of the sound pressure at the source position normalised by the square root of the spatial average of the mean square pressure (an independent, normally distributed variable with zero mean). The modification of x corresponds to a ‘modulation’ of the average mean square pressure due to the variation in the sound power emitted by the monopole that tends to increase the relative ensemble variance of the mean square pressure. Note that Eq. (8) is similar to Davy’s Eq. (6) although the ‘correction’ is somewhat smaller. If $K = 3$ is used in Davy’s expression rather than the value for oblique modes in a rectangular room it becomes $\varepsilon^2\left\{p_{\text{rms}}^2\right\} \simeq 1 + 6/M_{\text{s}}$, which is quite similar to Eq. (8).

One may perhaps expect Eq. (3) to be modified in a similar manner,

$$\varepsilon^2\left\{w_{\text{kin}}\right\} = \varepsilon^2\left\{w_{\text{tot}}\right\} = \frac{1}{3} + \frac{2F\left(M_{\text{s}}\right)}{3M_{\text{s}}}. \qquad (9)$$

Equations (8) and (9) are new and have not been validated.

3. Experimental results

To test Eqs. (6), (8) and (9) some experiments have been carried out in various rooms at the Technical University of Denmark: a small room (40 m^3) with bare walls, the same room with absorption added, a large reverberation room (245 m^3) with very little damping, and the same room with added absorption. Figure 1 shows the reverberation time of the rooms, measured in one-third octave bands using the interrupted noise method and a Brüel & Kjær (B&K) ‘PULSE’ analyser.

The monopole generating the sound field was a B&K OmniSource fitted with a ‘Volume velocity adapter’ with two matched quarter-inch microphones,[21] and the sound pressure and three perpendicular components of the particle velocity were measured using a ‘USP’ pressure-velocity probe produced by Microflown. The frequency response between the volume velocity of the source and the four signals from the USP device were measured with a same B&K analyser but in the FFT mode, using pseudorandom noise (6400 spectral lines) in the frequency range up to 3.2 kHz. A similar technique was used recently to validate Eqs. (5) and (7).[19] In order to approach the full variation associated with ensemble statistics both the source and the receiver positions were varied, and in the postprocessing of the results (obtained at 25 pairs of positions), additional variations over 8 Hz frequency bands (sixteen neighbouring frequencies) were also taken into account.

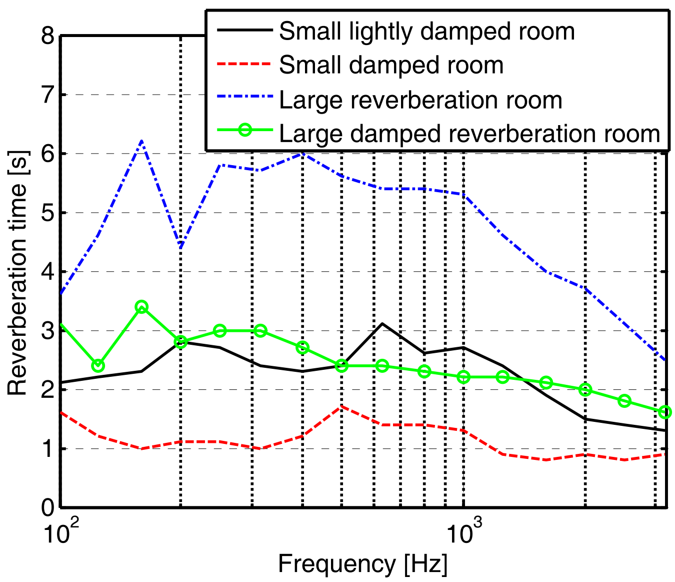

Figure 1. Reverberation time.

Figure 2 compares the results of the measurements of the mean square pressure with predictions calculated using Eq. (6) using a weighted average of K for oblique, tangential and axial modes as suggested by Davy,[14] a similar prediction using $K = 3$, and a prediction calculated using Eq. (8) and $F = 2$. As can be seen the relative space-frequency standard deviations of the mean square pressure in all rooms fluctuate somewhat with the frequency. Above 1 kHz they approach unity as expected, and at lower frequencies the standard deviations tend to be higher, in agreement with Eqs. (6) and (8). In general the experimental data seem to agree fairly well both with Eqs. (6) and (8). However, because of the significant fluctuations in the data it is not possible to conclude from these results whether the modal theory is improved with $K = 3$.

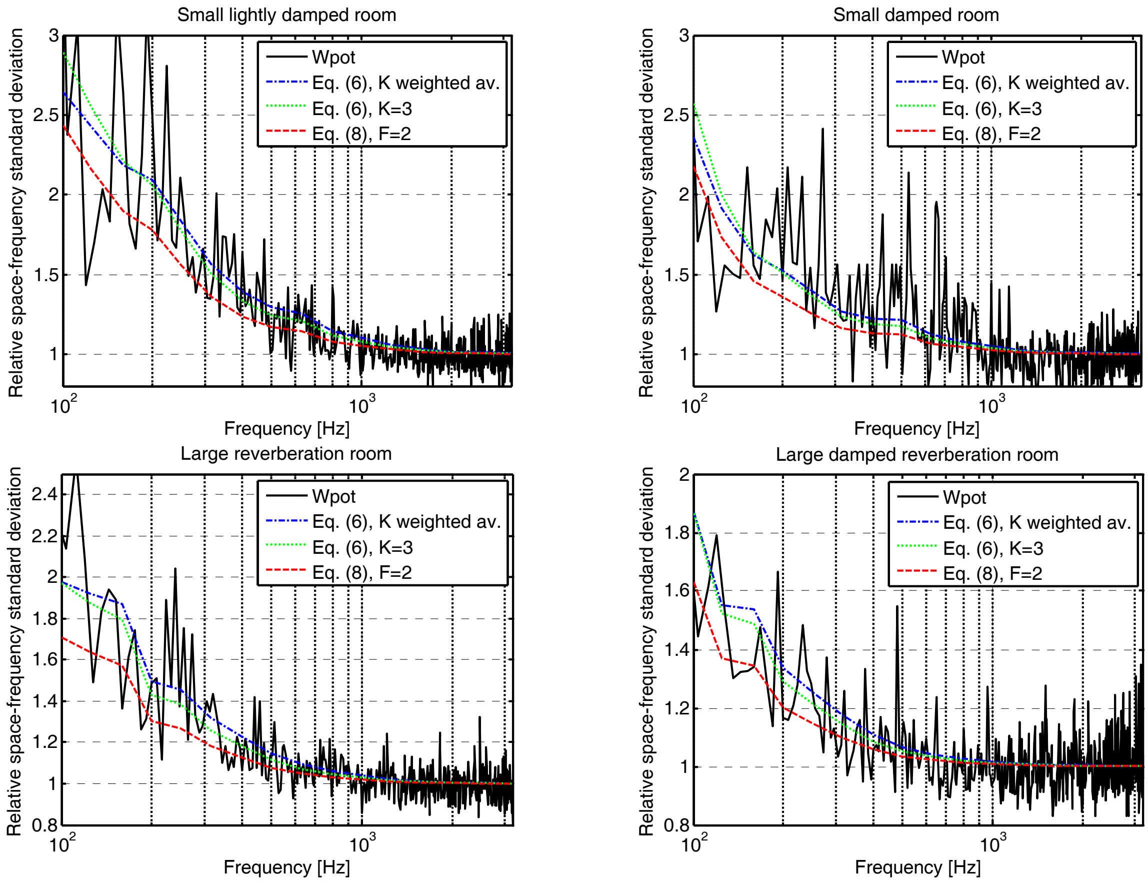

Figure 2. Relative space-frequency standard deviation of mean square pressure compared with predictions.

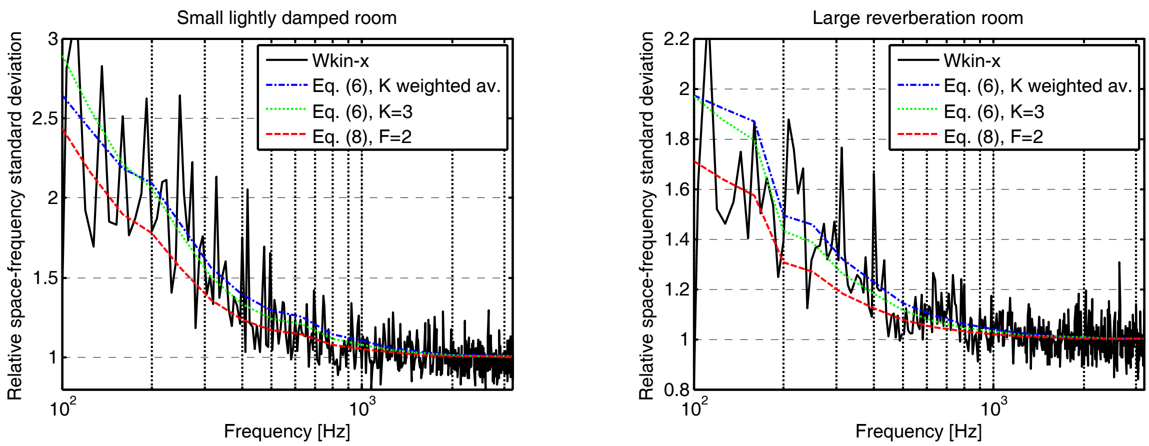

Figure 3. Relative space-frequency standard deviation of a mean square particle velocity component in an arbitrary direction compared with predictions.

Figure 3 shows the relative space-frequency standard deviation of a mean square particle velocity component in an arbitrary direction compared with predictions calculated using Eqs. (6) and (8). It is apparent that a single particle velocity component exhibits the same statistical behaviour as the mean square pressure.

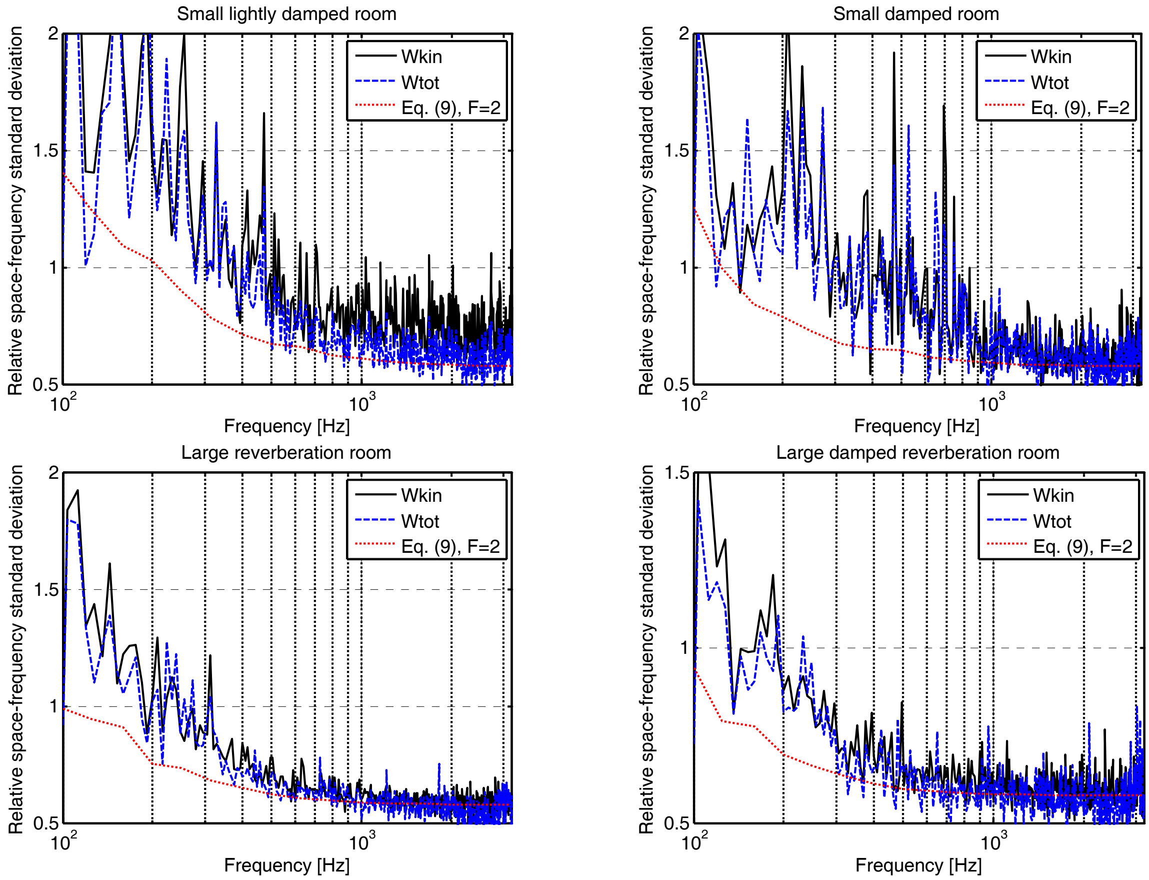

Figure 4. Relative space-frequency standard deviation of kinetic and total energy density compared with predictions.

Figure 4 compares the relative space-frequency standard deviation of kinetic energy density and total energy density with a prediction calculated using Eq. (9). It can be seen that whereas the observed standard deviations approach $1/\sqrt{3} \simeq 0.58$ above the Schroeder frequency as predicted,[9] they take higher values than predicted by Eq. (9) when the modal overlap is lower, even though the individual particle velocity components behave as predicted also at low frequencies as demonstrated by the results shown in Fig. 3. One possible explanation for this phenomenon might be that the three perpendicular mean square particle velocity components are not uncorrelated in this frequency range as assumed.

4. Numerical results

It is not practical to determine the full ensemble standard deviation experimentally, but it can be done with a numerical model. In this case the matter has been examined with two different numerical models, a boundary element (BEM) model and a finite element (FEM) model, but so far only for two-dimensional rooms. The dimensions of the rooms were chosen as uniform random variables varying between 2 and 6 m. The source position were placed at random but at least 0.4 m away from any wall. The FEM calculations were carried out from 200 Hz to 300 Hz, and the BEM calculations were carried out 200 Hz to 400 Hz, in both cases with a frequency step of 1 Hz. The element size was chosen so as to provide a low numerical pollution in the examined frequency range. For the FEM simulations the mean square pressure was taken at the nodal points of the mesh (around 6000 points); and for the BEM simulations the mean square pressure was calculated at 1000 random points in the room. In both cases, points or nodes closer than 0.4 m away from the walls or closer than 1 m from the source were not used. In order to determine the relative ensemble standard deviation as a function of the modal overlap the data were sorted into appropriate modal overlap intervals.

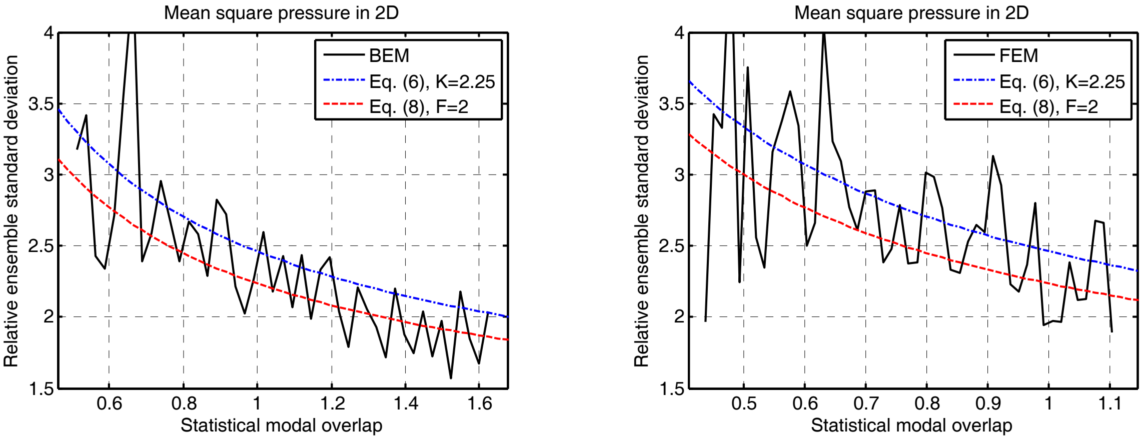

Figure 5. Relative ensemble standard deviation of mean square pressure in two-dimensional rooms compared with predictions.

In Fig. 5 the results of the numerical calculations are compared with predictions calculated using Eq. (8) and $F = 2$, and with a modified version of Eq. (6) more appropriate for low modal overlap[8,17] ($\varepsilon^2\left\{p^2_{\mathrm{rms}}\right\} = 1 + K^2/M_{\mathrm{s}}$) and $K = (3/2)^2$. The numerical data seem to agree equally well with the two predictions.

5. Discussion

Equation (7) is, arguably, a first order approximation that might be improved by taking account of the increased variance of the reverberant part of the sound pressure predicted by Eq. (8). However, the experimental and numerical results presented in Ref. 19 confirm Eq. (7) and do not support any such 'higher order correction'.

6. Conclusions

Experimental and numerical results confirm that there is a substantial increase in the relative ensemble variance of the mean square pressure in a reverberation room driven by a monopole that emits a pure-tone below the Schroeder frequency. Above this frequency the relative variance approach unity; below this frequency there is an increase in the variance that is inversely proportional to the modal overlap, that is, proportional to the ratio of the reverberation time to the room volume and inversely proportional to the square of the frequency.

Waterhouse's simple random wave theory has been extended to the frequency range below the Schroeder frequency and shown to give predictions of the relative ensemble variance of the mean square pressure in good agreement with the more complicated statistical modal theory due to Lyon, Davy and Weaver, and these predictions are confirmed by the experimental and numerical results.

The ensemble statistics of kinetic and total energy density has also been examined experimentally. Above the Schroeder frequency the relative ensemble variance of these quantities is predicted to be three less than the relative ensemble variance of the mean square pressure, and this is conformed. However, at lower frequencies the relative ensemble variance is larger than predicted, indicating that the advantage of measuring total energy density is reduced in this frequency range.

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