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FAR FIELD SOURCE LOCALIZATION USING TWO TRANSDUCERS: A "VIRTUAL ARRAY" APPROACH

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Virtual arrays can be created by using data acquired with scanning measurements techniques such as "Scan&Paint" [1–4]. This method is based on mixing tracking information with the signals recorded in order to characterise time stationary sound fields. With one transducer, variations of sound pressure or particle velocity can be assessed. Furthermore, relative phase of the sound field can be preserved by using an additional fixed reference sensor. Thereby, magnitude and phase information at discrete spatial areas are obtained, such as if the field were measured with a virtual array. The resolution of virtual arrays can be modified by changing the area which corresponds to a single data point. This property allows enhancing the spectral frequency range of conventional arrays.

This paper introduces the underlying theory of virtual arrays and validates the technique developed solving a problem which has high accuracy requirements: multiple source localization.

1. Introduction

There are many applications which require using transducer arrays in order to map magnitude and phase variations across the space. Traditionally, this fact directly implies investing huge amounts of money in an acquisition system. Furthermore, the resolution of the measurements would depend on the number of transducers used and their positions. If the array is constituted by too many sensors, it becomes acoustically significant, biasing the sound field aimed to characterise.

The following approach can be undertaken so as to avoid all constrains of conventional arrays if the sound field is assumed time stationary. Magnitude and phase of the sound field could be measured by using only two transducers, one situated in a fixed position and another moving across the measurable area. Tracking information is acquired by processing video information of the scanning sensor. Then, segments of a long time sequence can be evaluated for different areas in the space. Moreover, the relative phase of the sound field can be acquired calculating the phase differences between the two transducers. This powerful technique, which relies on using a "virtual array" can simplify many common problems due to its low time and cost requirements, but it also can improve the accuracy of traditional results due to its adaptable resolution and resizeable measurement area.

In the following sections not only the underlying theory is introduced but also some simulations and experiments are presented.

2. Theory

As a first approach, the sound field produced by a monopole source is studied. This approximation will give a good understanding of how the phase can be acquired at multiple points even measuring at different time instants by using only two transducers. Furthermore, the synthesis process required to generate the spectral matrix is introduced. Symmetrical and asymmetrical virtual arrays are studied.

2.1 Radiation from a monopole source

The acoustic source simplest to analyse is a sphere whose volume varies sinusoidally with time. While pulsating spheres are meaningless from a practical point of view, their theoretical features provides an ideal prototype for simplifying the behaviour of complex sources. In a medium that is infinite, homogeneous, and isotropic, a monopole source will produce an outgoing spherical wave [5]

$$\mathbf{p}(r,t) = \frac{\mathbf{A}}{r}\, e^{j(\omega t - kr)} \tag{1}$$

where $\mathbf{A}$ is determined by the source features, as can be seen below; k is the wavenumber; ω is the frequency of the sinusoidal movement and r is the distance from the centre of the sphere. Consider a sphere of radius a, vibrating radially with complex speed $U_o \exp(j\omega t)$, where the displacement of the surface is much less than the radius, $U_o/\omega << a$. The acoustic pressure of the fluid in contact with the sphere is given by Equation (1) evaluated at $r = a$. (This is consistent regarding small amplitude variations of linear acoustics). The pressure at the surface of the source can be defined as

$$\mathbf{p}(a,t) = Z_0 U_0 cos(\theta_a)\, e^{j(\omega t - ka + \theta_a)} \tag{2}$$

where Z_0 is the characteristic acoustic impedance of the medium evaluated ($\rho_0 c_0$); and $\theta_a = \coth(ka)$, so it is a function of the ratio of the source distance to the wavelength. Comparing Equation (1) and Equation (2) can be inferred that $\mathbf{A}$ is time and spatial independent, and it can be defined such as,

$$\mathbf{A} = Z_0 U_0 cos(\theta_a)\, e^{j(ka + \theta_a)} \tag{3}$$

Now, assessing the complex pressure over the sound field produced by the monopole source, since $\mathbf{A}$ will be independent of distance r, for time instance $t = 0$

$$\mathbf{p}(r) = \frac{\mathbf{A}}{r}\, e^{-jkr} \tag{4}$$

Therefore, if the phase response of the source is known, phase information can be predicted beforehand at any point by calculating the distance r_n between the evaluated position and the sound source.

2.2 Phase mapping

Absolute time phase measurements implies acquiring information at discrete points on the space simultaneously. Since the number of transducer used in scanning techniques is reduced due to the fact that they are moving during the measurement, absolute phase information cannot be characterised. Nevertheless, if the sound field can be assumed time stationary, relative phase variations can be measured at different time instances, allowing to use scanning techniques also for phase issues.

An additional fixed transducer is required in order to compare its phase information with the scanning sensor at the same time instance. Consequently, relative phase changes between two positions will be obtained. Because the scanning transducer is moved, phase variations can be characterised at any point in the space repeating the same procedure. One common way to obtain phase differences between two signals is by computing their cross-power spectral density [6]

$$\mathbf{P_n^{rel}}(\omega) = \lim_{T\to\infty} \frac{E\left\{P^*_{ref}(\omega)P_n(\omega)\right\}}{T} \tag{5}$$

where P^*_{ref} is the complex conjugate Fourier Transform of the time signal recorded by a pressure sensor at a fixed reference position and P_n represent each of the time sequences recorded at different position in the space. Moreover, the pressure generated by a monopole source at certain position given in Equation (1) can be evaluated now after computing the cross correlation between any two points. The last expression can be re-arranged so as to demonstrate that the new relative pressure is time independent, i.e.

$$\mathbf{P_n^{rel}}(\omega) = \frac{(Z_0U_0cos(\theta_a))^2}{r_n r_{ref}} e^{-jk(r_n - r_{ref})} \tag{6}$$

As can be seen in Equation (6), phase information of each point of the time stationary sound field is preserved in the exponential term, which only depends on the wavenumber and the distance difference between source and transducer positions.

In conclusion, it has been proved analytically that relative phase changes of a sound field can be mapped by taking the cross-correlation between two transducers: one at a fixed position and the other scanning the positions aimed to measured.

2.3 Spectral matrix

The spectral matrix is the key of powerful spectral estimation techniques such as Capon or MUSIC. Eigenvalue decomposition of the spectral matrix is a way of measuring the number of independent components that constitute a signal. This can be used to distinguish between signal subspace (high eigenvalues) and noise subspace (low eigenvalues).

Commonly, the spectral matrix is calculated using time data from array sensors. Nonetheless, since all information cannot be acquired at different positions simultaneously using “virtual arrays”, a different approach has to be implemented. Measuring pressure at n positions and cross-correlating the data with a fixed sensor we can define the Fourier Transform matrix of the results as

$$\mathbf{p_n^{rel}}(\omega) = [p_1^{rel} \;\; p_2^{rel} \;\; ... \;\; p_n^{rel}] \tag{7}$$

Then, the spectral matrix of the relative complex pressures measured can be defined such as,

$$\begin{aligned} \mathbf{C^{rel}}(\omega) &= E\left[\mathbf{p_n^{rel}}(\omega)\left(\mathbf{p_n^{rel}}(\omega)\right)^H\right] = \\ &= \begin{bmatrix} p_1^{rel}(\omega)\overline{p_1^{rel}(\omega)} & ... & p_1^{rel}(\omega)\overline{p_n^{rel}(\omega)} \\ \vdots & \ddots & \vdots \\ p_n^{rel}(\omega)\overline{p_1^{rel}(\omega)} & ... & p_n^{rel}(\omega)\overline{p_n^{rel}(\omega)} \end{bmatrix} \end{aligned} \tag{8}$$

where the operator ^{H} denotes the complex conjugate transpose. According to Equation (6) and Equation (8), the spectral matrix produced by a single monopole source in the free field can be defined then such as,

$$\mathbf{C^{rel}} = \frac{(Z_0 U_0 cos(\theta_a))^4}{r_{ref}^2} \begin{bmatrix} \frac{e^{jk(r_1-r_1)}}{r_1^2} & \cdots & \frac{e^{jk(r_1-r_n)}}{r_1 r_n} \\ \vdots & \ddots & \vdots \\ \frac{e^{jk(r_n-r_1)}}{r_n r_1} & \cdots & \frac{e^{jk(r_n-r_n)}}{r_n^2} \end{bmatrix} \tag{9}$$

The phase of the spectral matrix $\mathbf{C_{rel}}$ can be expressed as a product of the wavenumber k and the separation difference between the noise source and any position measured,

$$\angle\mathbf{C^{rel}} = k \begin{bmatrix} (r_1 - r_1) & \dots & (r_1 - r_n) \\ \vdots & \ddots & \vdots \\ (r_n - r_1) & \dots & (r_n - r_n) \end{bmatrix} \tag{10}$$

As it has been point out above, the spectral matrix is calculated conventionally from time data of the different elements of the array. This will create a matrix with a maximum rank that depends on the number of sensors. However, the rank of matrix $\mathbf{C_{rel}}$ is, by definition, constrained to unity since the matrix has been created by combinations of only one linearly independent function made up from complex pressure values at a certain frequency. This fact will imply that most of the information of the spectral matrix has been missed. Only one eigenvalue will be high, regardless of the number of uncorrelated signals which create the sound field, as if one sound source were creating the entire sound field. So to overcome rank problems, two different approaches have been studied: measuring the noise sources individually and performing the measurements under far field conditions for reconstructing the spectral matrix taking advantage of its intrinsic symmetry.

The first solution proposed guarantees a perfect reconstruction of the conventional $\mathbf{C}(\omega)$ from individual relative spectral matrices exciting the sound field with only one noise source at a time, i.e.

$$\mathbf{C}(\omega) = \sum_{s=1}^{L} \mathbf{C_s^{rel}}(\omega) \tag{11}$$

where L is the number of noise sources which produce the sound field. Despite the fact that the spectral matrix is calculated accurately, this method does not suit all possible practical scenarios.

Secondly, if far field conditions are satisfied, the spectral matrix $\mathbf{C}(\omega)$ is guaranteed to be hermitian as a consequence of equation (10). Figure 1 presents an 3x3 array using two reference sensors at the top corners and its corresponding spectral matrix. The arrows between the positions represent the transfer function between two elements. On the right hand side, each term of the spectral matrix is plotted with a colour. As can be seen, the matrix follows a simple pattern. Therefore, it is possible to synthesize the whole matrix by using only elements of the two rows and columns highlighted.

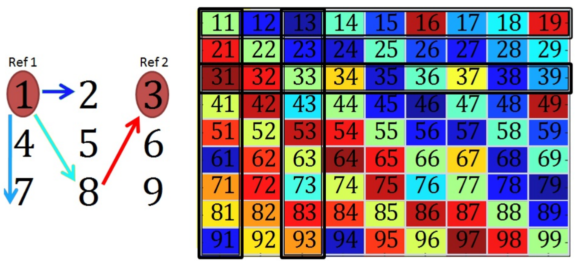

Figure 1. Sketch of a 3x3 array and its corresponding spectral matrix

A relative spectral matrix $\mathbf{C^{rel}}(\omega)$ can be obtained using any sensor as a reference. This matrix will have common elements with $\mathbf{C}(\omega)$ at the row and column where the reference sensor was situated. Consequently, if $\mathbf{C^{rel}}(\omega)$ is calculated twice but using two different reference sensors at the top corners of the array, all elements required for reconstructing $\mathbf{C}(\omega)$ (highlighted elements on Figure 1) will be found. In conclusion, if measurements are undertaken under far field conditions with two reference sensors even the whole spectral matrix can be reconstructed accurately.

In addition, asymmetrical array geometries can be implemented by calculating first a $\mathbf{C}(\omega)$ of an rectangular or square array which contains the asymmetric version aimed to emulate. Then, unwanted elements could be directly removed from the spectral matrix by removing their corresponding row and column.

2.4 Source localization and DOA algorithms

One common application for sensor arrays is to determine the direction of arrival (DOA) of propagating wavefronts. In this section two different techniques are assessed: conventional sum and delay beamforming and Multiple Signal Classification (MUSIC) applied to source localization.

2.4.1 Conventional Beamforming

Generally, an array receives spatially propagating signals and processes them to estimate their direction of arrival; it acts as a spatially discriminating filter [7]. This spatial filtering operation is known as beamforming. An conventional array processor steers a beam to a particular direction by computing a properly weighted sum of the individual sensor signals. Thus, this procedure results in the coherent addition of signals coming from the direction of focus, maximising the energy in the beamformer output, whereas signals from other directions will be attenuated.

For short or intermediate distance away from a noise source, spherical propagation of the wavefront given in Equation (4) should be taken into account. If the sound field is produced by a simple source, waves arriving at the array can be characterised by

$$B(\omega) = \frac{1}{L}\sum_{n=1}^{L} p_n^{rel}(\omega)e^{-j\omega\tau_n} = \frac{1}{L}\sum_{n=1}^{L} p_n^{rel}(\omega)e^{-jk\,r_n}$$

where τ_n is the delay necessary to apply to the signal recorded by the L transducers for focusing the beam towards the source. If the array is sufficiently far from the source, the resulting wavefronts sampled by the array may be regarded as plane waves. In this case, τ_n can be defined as a function of a unit vector $\hat{\gamma}$ related to the position of the transducer $\vec{x}$,

$$\tau_n = \frac{\hat{\gamma}\cdot\vec{x}}{c} \tag{12}$$

Therefore, evaluating Equation (12) for far field conditions leads to

$$B_{ff}(\omega) = \frac{1}{L}\sum_{n=1}^{L} p_n^{rel}(\omega)e^{-j\hat{\gamma}\cdot\vec{x}} \tag{13}$$

2.4.2 Multiple Signal Classification

MUSIC is an acronym which stands for Multiple Signal classification. It is a high resolution technique based on exploiting the eigenstructure of the spectral matrix [8]. If D uncorrelated signals are impinging on L elements of an array, the eigenstructure of the spectral matrix can be used to distinguish between signal subspace, defined by D high eigenvalues; and noise subspace, constituted by $L-D$ low energy components. Then, the spectral matrix can be divided into two different terms, i.e.

$$\mathbf{C}(\omega) = A\,C_{SS}\,A^H + \sigma_n^2 I \tag{14}$$

where $A = [a(\sigma_1), a(\sigma_2), a(\sigma_3), ...a(\sigma_D)]$ is $L \times D$ array steering matrix; σ_n^2 is the noise variance and $C_{SS}(\omega) = [s_1(k), s_2(k), s_3(k), ...s_D(k)]$ is $D \times D$ source spectral matrix. $\mathbf{C}$ has D eigenvectors associated with signals and $L - D$ eigenvectors associated with the noise. Hence, we can then construct the $L \times (L - D)$ subspace spanned by the noise eigenvectors such that

$$V_N = [v_1, v_2, v_3, ...v_{L-D}] \tag{15}$$

The noise subspace eigenvectors are orthogonal to array steering vectors at the angles of arrivals $\sigma_1, \sigma_2, \sigma_3, \sigma_D$. Consequently, the MUSIC Pseudospectrum is given as

$$P_{MUSIC}(\sigma) = \left|a(\sigma)^H V_N V_N^H a(\sigma)\right|^{-1} \tag{16}$$

3. Simulations

For techniques which require use the spectral matrix, it has to be synthesized assuming far field conditions for handling a symmetric matrix. There is an inherent positional error in the method used. This error will depend on the distance from the source. Figure 2 shows the error variation by changing separation between the source and the array.

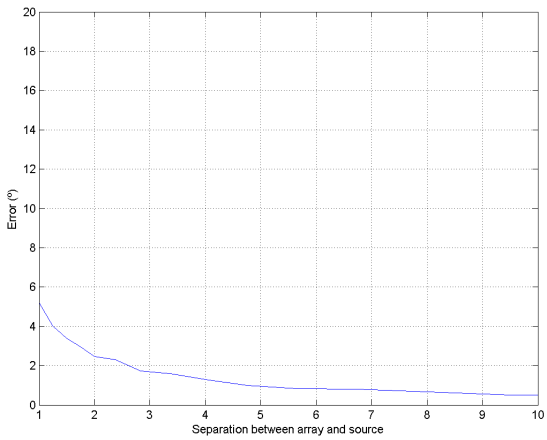

Figure 2. MUSIC positional error after synthesize the spectral matrix for different separation distance r_n

4. Instrumentation and measurement setup

A source localization experiment has been undertaken in a small anechoic chamber of the Institute of Sound and Vibration (UK). Two loudspeakers KEF KHT 3005 have been localised by scanning an area of 0.5x0.35 m. The loudspeakers have been excited with broadband white noise. In addition, two reference microphones have been allocated at the left and right top corners of the virtual measurement plane.

5. Results and discussion

Three different scans of 3 minutes each have been undertaken. Positions with less than three sweeps within a block have been removed in order to avoid poor spectral estimations. So, 118 virtual elements have been used to generate the results presented in this section.

5.1 Conventional Beamforming

A conventional beamforming (CBF) algorithm has been implemented according to Equation (12). First of all, Figure 3 shows on the left hand side an error graph between theoretical and estimated noise source positions for different frequencies. As can be seen, CBF works very stable along the spectral range evaluated. Low variance results which guarantee good position estimation with fairly constant bias across the spectrum. It is also important to highlight that localization is preserved even at very high frequencies such as 8 kHz, when the width of each area of the grid (0.03 m) becomes comparable to the wavelength (< 0.05 m) and small errors become significant.

In addition, a localization map at 5 kHz is presented on the right hand side of Figure 3. The huge number of sensors emulated (118) produce a very clear picture with two clear maxima. Assessing how localization maps change across frequency, the width of the lobes decrease significantly at high frequencies whereas the noise increases. This is because the number of wavelengths at the measured plane increases with frequency, leading to sharper graphs. Meanwhile, as the areas of the grid become significant, noise artefacts become significant.

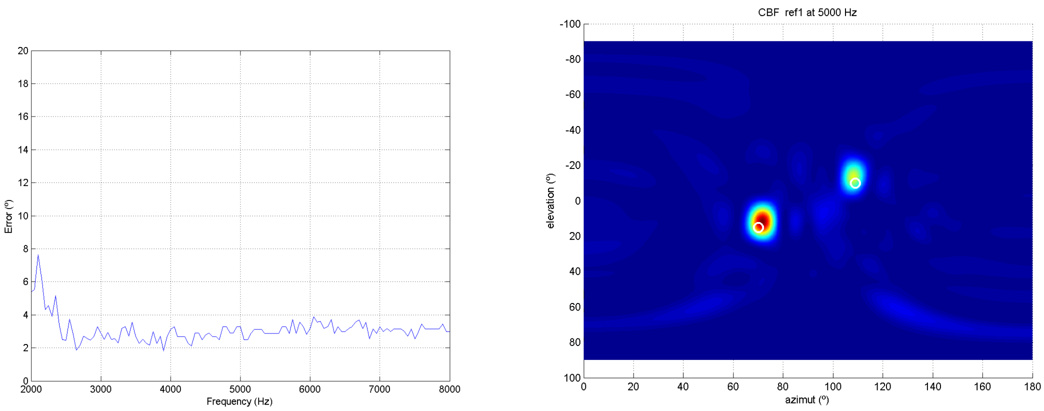

Figure 3. CBF error across the spectrum (left) and CBF localization map at 5 kHz (right)

5.2 MUSIC

This parametric spectral estimation technique has been implemented following Equation (16) for source localization purposes. As explained in Section 2.4.2, MUSIC relies on eigenvalue decomposition of the spectral matrix of the data. Ideally, the eigenspectrum will follow the same trend for the whole frequency range if, for example, the sound field is produced by two identical incoherent sources. Nonetheless, in practise, spectral estimation is never exact and small errors will become significant. Figure 4 presents the error of the measurements across the spectrum (left) and sound localization map for 5 kHz (right). As can be seen the peaks are sharper than with CBF and the signal to noise ratio is also greater. However, MUSIC behaves less stable across the spectrum, although, on average it will give the same result with better resolution. Thereofore, superresolution is reached at the cost of higher sensitivity to noise.

6. Conclusions

A novel technique for mapping phase variations across a time stationary sound field has been introduced. Not only the theoretical bases have been presented but also, experimental data has been shown. The MUSIC methods promises superresolution at the cost of higher sensitivity to noise. Conventional beamforming has lead to superior results in this framework, possibly due to the lower sensitivity to the approximations which are inherently present in the method.

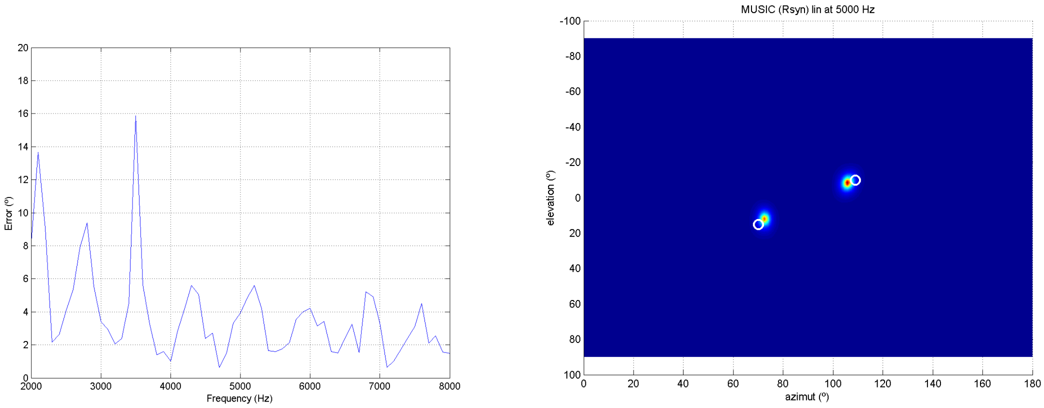

Figure 4. MUSIC error across the spectrum (left) and MUSIC localization map at 5 kHz (right)

Results obtained probe the reliability of the proposed method which has a series of advantages compared to using conventional arrays. Firstly, the cost of two or three transducer (depending of the localization technique used) compared with 118 which has been emulated is a huge difference. Also, due to the adaptive resolution, a greater frequency range can be assessed. Moreover, high density microphone arrays will acoustically bias the results significantly, whereas “virtual arrays” do not. Also, the flexibility of the method allows to easily rescale the size of the plane measured from a couple of centimetres to several meters just by moving the camera used to acquire the tracking information.

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